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Polarization Density Matrix for Heavy $Q\bar{Q}$ Production in Hadron Colliders

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Abstract

We derive the polarization density matrix for heavy quark-antiquark pair produced by gluon fusion or quark annihilation. These formulas are applied to find the angular correlations of the W^+W^- pairs from ultra-heavy $Q\bar{Q}$ decays.

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The heavy flavor production has been of constant interest in the past decade [1–18]. The focus has shifted from charm and bottom to top and to ultra–heavy quarks in connection with a possible fourth generation or other extensions beyond standard model. One of the first theoretical efforts has always been to estimate the total and differential production cross sections. The information contained in the polarizations of the produced quarks are supposed to be lost as they fragmented into mesons and then decayed. While this is probably a good approximation for charm and bottom, it had been argued that for top and for ultra–heavy quarks the polarizations can make a significant difference to the calculated decay correlations[11].

There is no doubt that finding the top and measuring its mass accurately is one of the major experimental concerns today. However, it has been pointed out that the existence of ultra–heavy quarks with masses of the order of 100 GeV may well be probed in the present hadron colliders [12,13,15]. Indeed, with an estimated $Q\bar{Q}$ production cross section of the order of 100 pb for quark mass around 100 GeV at Tevatron[13], even a study of decay correlations is feasible.

In this note, we give the polarization density matrix for $Q\bar{Q}$ produced by gluon fusion and by $q\bar{q}$ annihilation. The corresponding quantity for $Q\bar{Q}$ produced in e^+e^- collisions was given by Anselmino, et al.[14].

The polarization density matrix can be used in a number of ways. In particular, we shall use them to give the decay correlations of the W-pairs for the case of ultra–heavy $Q\bar{Q}$ production. The effect of including polarization correlation is found to be of the order of ten percent for quark mass m in the range $m_w < m < 2m_w$ in this particular situation.

Let us consider first the case of gluon fusion and denote by $M\,(p_1\sigma_1, p_2\sigma_2; q_1\lambda_1, q_2\lambda_1)$ the amplitude for two gluons with four-momenta p_1, p_2 and helicity σ_1, σ_2 to fuse into $Q\bar{Q}$ of momenta q_1, q_2 and helicity λ_1, λ_2 respectively. The polarization density matrix $\mathbf{P}$ is defined by

$$\mathbf{P}_{\lambda_1'\lambda_1;\lambda_2\lambda_2'} = \frac{1}{4(N^2-1)^2} \sum_{\sigma_1\sigma_2} M\,(p_1\sigma_1, p_2\sigma_2; q_1\lambda_1, q_2\lambda_2)\, M^*\,(p_1\sigma_1, p_2\sigma_2; q_1\lambda_1', q_2\lambda_2') \tag{.1}$$

where N is the number of colors.

For the helicity eigenstates of massive quarks, we use the convention of Ref. 19.

These helicity eigenstates can be shown to satisfy the relations

$$u(p,\lambda)\,\bar{u}(p,\lambda') = \frac{1}{2}(m+\not p)\left(\not e^0\sigma_0 - \not e^i\gamma_5\sigma_i\right)_{\lambda\lambda'} \quad (.2)$$

$$v(p,\lambda)\,\bar{v}(p,\lambda') = \frac{1}{2}(m-\not p)\left(\not e^0\bar\sigma_0 + \not e^i\gamma_5\bar\sigma_i\right)_{\lambda\lambda'} \quad (.3)$$

Where $\sigma_0 = 1$ is the 2x2 identity matrix, σ_i are the usual Pauli matrices, $\bar\sigma_a = \sigma_2\sigma_a\sigma_2$ and e^a are the following four vectors:

$$e^0(p) = \frac{1}{m}p \qquad m^2 = p^2 \quad (.4)$$

$$e^1(p) = \frac{1}{|p|(|p|+p_z)} < 0,\ -p_z(|p|+p_z) - p_y^2, p_x p_y, p_x(|p|+p_z) > \quad (.5)$$

$$e^2(p) = \frac{1}{|p|(|p|+p_z)} < 0,\ -p_x p_y, p_z(|p|+p_z) + p_x^2,\ -p_y(|p|+p_z) > \quad (.6)$$

$$e^3(p) = -\frac{1}{m} < |p|,\ \frac{p^0}{|p|}\vec{p} > \quad (.7)$$

In the above formulas, $|p|$ is the absolute value and p_x, p_y, p_z are the components of the three-momentum $\vec{p}$. When $\vec{p}$ approaches the direction of the negative Z-axis, we choose the convention to put $p_y = 0$ first. We also note that $e^\mu_\nu(p)$ is a matrix for Lorentz transformation.

It is easy to prove the following useful identities:

$$\gamma_\mu u\bar{u}\gamma^\mu = 2m\sigma_0 - m(\not e^0\sigma_0 - \not e^i\gamma_5\sigma_i) \quad (.8)$$

$$\gamma_\mu v\bar{v}\gamma^\mu = -2m\bar\sigma_0 - m(\not e^0\bar\sigma_0 - \not e^i\gamma_5\bar\sigma_i) \quad (.9)$$

Using equations (2),(3),(8) and (9), we obtain the density matrix $\mathbf{P}$ for gluon fusion.

$$\mathbf{P} = \frac{4\pi^2\alpha_s^2}{N}\left(x - 1 - \frac{1}{N^2-1}\right)\tilde{\mathbf{P}} \quad (.10)$$

$$\begin{aligned}\tilde{\mathbf{P}} &= \sigma_0\otimes\bar\sigma_0 + \left[-\frac{1}{x} + \frac{1}{\gamma^2}\left(1 - \frac{x}{2\gamma^2}\right)\right]\left(\sigma_0\otimes\bar\sigma_0 + e_1^i\cdot e_2^j\sigma_i\otimes\bar\sigma_j\right)\\ &+ \left(1 - \frac{x}{2\gamma^2}\right)\Big[\ \frac{1}{s}\left(p\cdot e_1^i p\cdot e_2^j - r\cdot e_1^i r\cdot e_2^j\right)\\ &+ \frac{t-u}{s^2}\left(r\cdot e_1^i p\cdot e_2^j - p\cdot e_1^i r\cdot e_2^j\right)\Big]\sigma_i\otimes\bar\sigma_j\end{aligned} \quad (.11)$$

where $\alpha_s = g^2/4\pi$ is the strong coupling constant, $e_1^i = e^i(q_1)$, $e_2^i = e^i(q_2)$ and

$$x = \frac{s^2}{2(t-m^2)(u-m^2)} \tag{.12}$$

$$\gamma^2 = (1-\beta^2)^{-1} = \frac{s}{4m^2} \tag{.13}$$

$$p = p_1 + p_2, r = p_1 - p_2, s = p^2, t = (p_1 - q_1)^2, u = (p_1 - q_2)^2 \tag{.14}$$

In the CM frame and choosing the positive Z-axis to be the direction of motion of the heavy quark Q, the X-Z plane to be the interaction plane so that

$$r = s^{\frac{1}{2}}(0, \sin\theta, 0, \cos\theta) \tag{.15}$$

$$x = \frac{2}{1-\beta^2\cos^2\theta} \tag{.16}$$

we can write $\tilde{\mathbf{P}}$ as

$$\begin{aligned} \tilde{\mathbf{P}} &= a_0\sigma_0 \bigotimes \bar{\sigma}_0 + a_1\sigma_1 \bigotimes \bar{\sigma}_1 + a_2\sigma_2 \bigotimes \bar{\sigma}_2 + a_3\sigma_3 \bigotimes \bar{\sigma}_3 \\ &+ a_4\left(\sigma_3 \bigotimes \bar{\sigma}_1 + \sigma_1 \bigotimes \bar{\sigma}_3\right) \end{aligned} \tag{.17}$$

where

$$a_0 = 1 - \frac{1}{x} - \frac{1}{\gamma^2}\left(1 - \frac{x}{2\gamma^2}\right) \tag{.18}$$

$$a_1 = a_0 - 1 + \left(1 - \frac{x}{2\gamma^2}\right)\sin^2\theta \tag{.19}$$

$$a_2 = 1 - a_0 \tag{.20}$$

$$a_3 = -a_0 + \left(1 - \frac{x}{2\gamma^2}\right)\left(1 + \cos^2\theta\right) \tag{.21}$$

$$a_4 = \frac{1}{\gamma}\left(1 - \frac{x}{2\gamma^2}\right)\sin\theta\cos\theta \tag{.22}$$

Note that

$$1 - \frac{x}{2\gamma^2} = \frac{\beta^2\sin^2\theta}{1-\beta^2\cos^2\theta} \tag{.23}$$

vanishes at threshold and in the forward/backward directions.

The differential production cross sections is proportional to TrP.

$$\frac{1}{4}Tr\mathbf{P} = \frac{4\pi^2\alpha_s^2}{N}\left(x - 1 - \frac{1}{N^2-1}\right)a_0 \qquad (.24)$$

which agrees with the known results [3-8] although it is in the simpler factorized form of Ref. 17.

For the density matrix $\mathbf{P}_q$, corresponding to the process $q\bar{q} \to Q\bar{Q}$, we obtain

$$\mathbf{P}_q = \frac{4\pi^2\alpha_s^2}{N}\left(\frac{N^2-1}{2N}\right)\tilde{\mathbf{P}}_q \qquad (.25)$$

$$\begin{aligned}\tilde{\mathbf{P}}_q &= \sigma_0 \bigotimes \bar{\sigma}_0 - \frac{1}{x}\left(1 - \frac{x}{2\gamma^2}\right)\left(\sigma_0 \bigotimes \bar{\sigma}_0 + e_1^i \cdot e_2^j \sigma_i \bigotimes \bar{\sigma}_j\right) \qquad (.26)\\ &+ \left[\frac{1}{s}\left(p \cdot e_1^i p \cdot e_2^j - r \cdot e_1^i r \cdot e_2^j\right) + \frac{t-u}{s^2}\left(r \cdot e_1^i p \cdot e_2^s - p \cdot e_1^i r \cdot e_2^i\right)\right]\sigma_i \bigotimes \bar{\sigma}_j\end{aligned}$$

In the CM frame and choosing the same convention for x and z axis as before so that equations (12),(13) are replaced by

$$r = s^{\frac{1}{2}}\beta_i\,(0, \sin\theta, \cos\theta) \qquad (.27)$$

$$x = \frac{2}{1 - \beta^2\beta_i^2\cos^2\theta} \qquad (.28)$$

where β_i is the CM velocity of the incoming quark, we find that

$$\begin{aligned}\tilde{\mathbf{P}}_q &= b_0\sigma_0 \bigotimes \bar{\sigma}_a + b_1\sigma_1 \bigotimes \bar{\sigma}_1 + b_2\sigma_2 \bigotimes \bar{\sigma}_2 + b_3\sigma_3 \bigotimes \bar{\sigma}_3\\ &+ b_4\left(\sigma_3 \bigotimes \bar{\sigma}_1 + \sigma_1 \bigotimes \bar{\sigma}_3\right) \qquad (.29)\end{aligned}$$

$$b_0 = 1 - \frac{1}{x} + \frac{1}{2\gamma^2} \qquad (.30)$$

$$b_1 = b_0 - 1 + \beta_i^2\sin^2\theta \qquad (.31)$$

$$b_2 = 1 - b_0 \qquad (.32)$$

$$b_3 = -b_0 + \left(1 + \beta_i^2\cos^2\theta\right) \qquad (.33)$$

$$b_4 = \frac{1}{\gamma}\beta_i^2\sin\theta\cos\theta \qquad (.34)$$

Again, the differential cross section for $Q\bar{Q}$ production is proportional to $Tr\mathbf{P}_q$ and our result agrees with the known one [4-8].

We can apply our formulas for polarization density matrix to find the decay correlations of W^+W^- pairs when the mass m of Q is large enough. Let $M(q_1,\lambda_1;k_1\sigma_1,r_1\tau_1)$ be the amplitude for the decay of Q into a W with four-momentum k_1, helicity σ_1, and a light quark of four-momentum r_1, helicity τ_1. We define the decay matrix $D_{\lambda\lambda'}$ to be

$$D_{\lambda\lambda'} = \sum_{\sigma_1\tau_1} M(q_1\lambda;k_1\sigma_1,r_1\tau_1)\, M^*(q_1\lambda';k_1\sigma_1,r_1\tau_1) \qquad (.35)$$

Similarly, let $\bar{M}(q_2\lambda_2;k_s\sigma_2,r_2\tau_2)$ be the decay amplitude for $\bar{Q}$ and define

$$\bar{D}_{\lambda'\lambda} = \sum_{\sigma_2\tau_2} \bar{M}(q_2\lambda;k_2;k_2\sigma_2,r_2\tau_2)\, \bar{M}^*(q_2\lambda';k_2\sigma_2,r_2\tau_2) \qquad (.36)$$

It is easy to show that

$$D = m^2|g_{Qqw}|^2 V_1\cdot e^a\sigma_a \qquad (.37)$$

$$\bar{D} = m^2|q_{\bar{Q}\bar{q}W}|^2 V_2\cdot e^a\sigma_a \qquad (.38)$$

where $V_{1,2}$ are the four vectors

$$V_i = c_i\hat{q}_i + c_i'\hat{l}_i$$

$$l_i = [k_i - (k_i\cdot\hat{q}_i)\,\hat{q}_i]\,,\quad \hat{l}_i = l_i/(-l_i^2)^{\frac{1}{2}} \qquad (.39)$$

$$c_i = \frac{1}{2}\left(\delta_i^2 - 3\frac{m_w}{m}\delta_i + 2\right) \qquad (.40)$$

$$c_i' = \frac{1}{2}\left(\delta_i - 3\frac{m_w}{m}\right)\left(\delta_i^2 - 4\right)^{\frac{1}{2}} \qquad (.41)$$

$$\delta_i = 2\hat{k}\cdot\hat{q}_i = \frac{1}{mm_w}\left(m^2 + m_w^2 - m_i^2\right) \qquad (.42)$$

In equation (42), m_i are the masses of the final state quarks. If we neglect the masses $m_{1,2}$, we have

$$c_1 = c_2 = \frac{1}{2m^2m_w^2}\left(m^2 + 2m_w^2\right)\left(m^2 - m_w^2\right) \qquad (.43)$$

$$c_1' = c_2' = \frac{1}{2m^2m_w^2}\left(m^2 - 2m_w^2\right)\left(m^2 - m_w^2\right) \qquad (.44)$$

Define the decay correlation matrix $\mathbf{D}$ by

$$\mathbf{D} = D \bigotimes \bar{D} \tag{.45}$$

Then the differential cross section for the production and decay of $Q\bar{Q}$ in the narrow width approximation is

$$d\sigma = \frac{1}{128\pi^3 s} \frac{Tr\,(\mathbf{PD})}{Tr\mathbf{D}} B\,(Q \to Wq)\, B\left(\bar{Q} \to W\bar{q}\right) d\,(\beta \cos\theta)\, d\Omega_1, d\Omega_2 \tag{.46}$$

where $B(Q \to Wq)B(\bar{Q} \to W\bar{q})$ are the branching ratios for $Q, \bar{Q}$ to decay to $Wq, W\bar{q}$ respectively and $d\Omega_1, d\Omega_2$ are the differential angular elements for W's in the respective rest frames of $Q, \bar{Q}$. In these frames,

$$\hat{l}_1 = (0, \sin\theta_1 \cos\phi_1, \sin\theta_1 \sin\phi_1, cos\theta_1) \qquad \hat{l}_2 = (0, \sin\theta_2 \cos\phi_2, \sin\theta_2 \sin\phi_2, \cos\theta_2) \tag{.47}$$

and we find

$$\begin{aligned} \frac{Tr(\tilde{\mathbf{P}}\mathbf{D})}{Tr\mathbf{D}} = a_0 \quad &+ \quad c\,\{\,(a_1 \cos\phi_1 \cos\phi_2 + a_2 \sin\phi_1 \sin\phi_2) \sin\theta_1 \sin\theta_2 \\ &+ \quad a_3 \cos\theta_1 \cos\theta_2 + a_4 (\sin\theta_1 \cos\theta_2 \cos\phi_1 + \cos\theta_1 \sin\theta_2 \cos\phi_2)\,\} \end{aligned} \tag{.48}$$

where

$$c = c_1' c_2' / c_1 c_2 \tag{.49}$$

The above formulas are for gluon fusion. For $q\bar{q}$ annihilation, we need only replace $a_0, ..., a_4$ by $b_0, ..., b_4$.

The effect of considering nontrivial polarization density matrix is measured by the magnitude of c and those of $a_i/a_0 \ \ (b_i/b_0)$. As can be seen from equations (43) and (44), c is nonnegative and is smaller than one. It is no larger than 12% for m between m_w and $2m_w$. c is, of course process dependent. a_i/a_0 do not depend on particular decay processes but only on β, the CM velocity of Q, and on θ, the CM production angle. Near threshold, $|a_i/a_o| \sim 1$ and $|b_i/b_0| < 1$. In general, their magnitude is of order 1 so that for $m_w < m < 2m_w$ we do not expect much more than 12% effect in the W^+W^- decay correlation.

In Fig. 1, we plotted c as a function of m/m_w. In Fig. 2, we plotted a_i/a_0 at $\theta = \pi/2$ as a function of β and Fig. 3 is a similar plot for b_i/b_0.

To obtain quantitative physical predictions for hadron colliders, we have to fold in equation (46) the parton distribution functions [20] and compute various interesting observables such as invariant mass or p_t distributions of the W-pairs. These investigations are in progress.

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Figure Captions

Fig. 1: C as a function of m/m_w.

Fig. 2: a_i/a_o at $\theta = \pi/2$ as a function of β.

Fig. 3: b_i/b_o at $\theta = \pi/2$ as a function of β.

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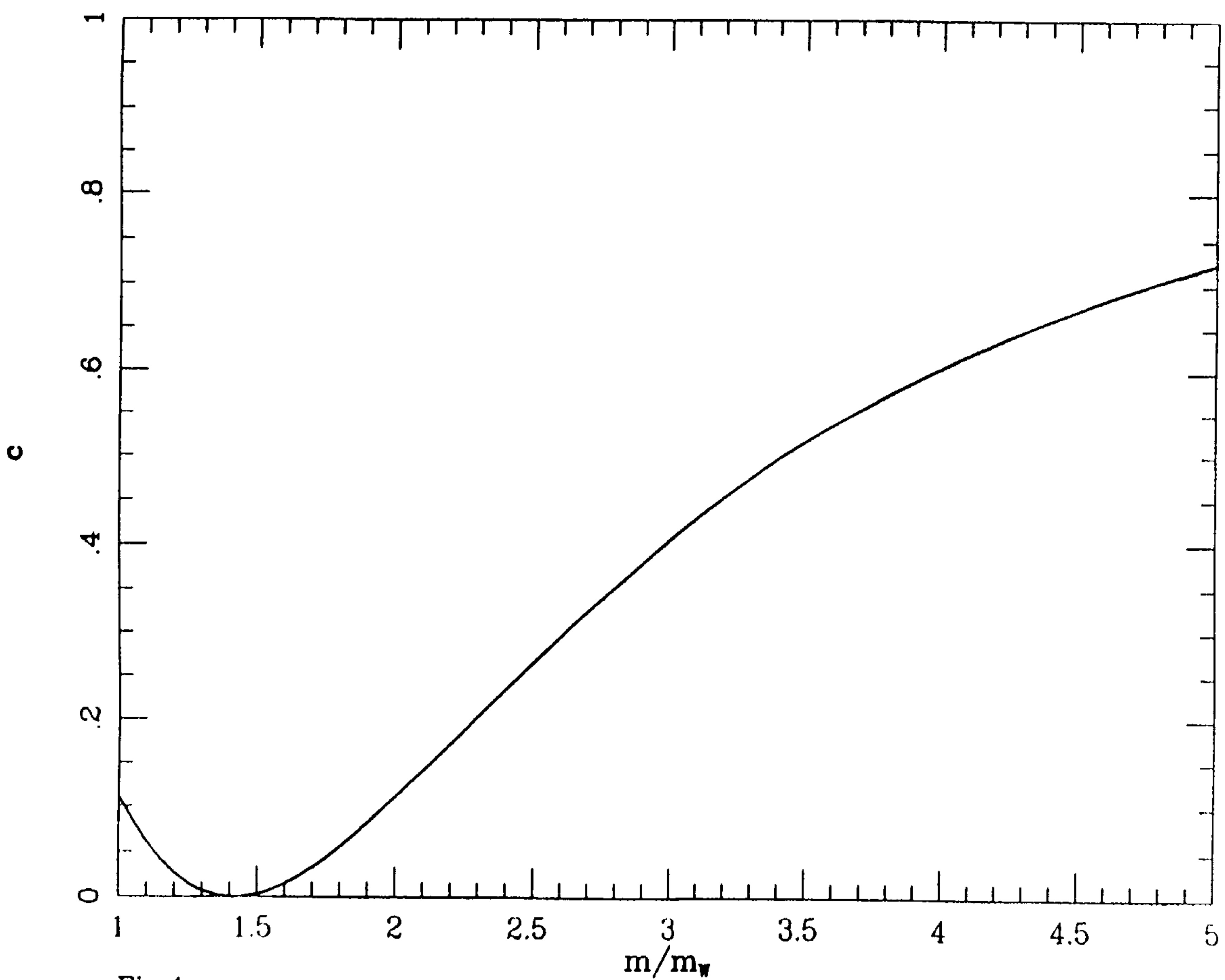

Fig.1

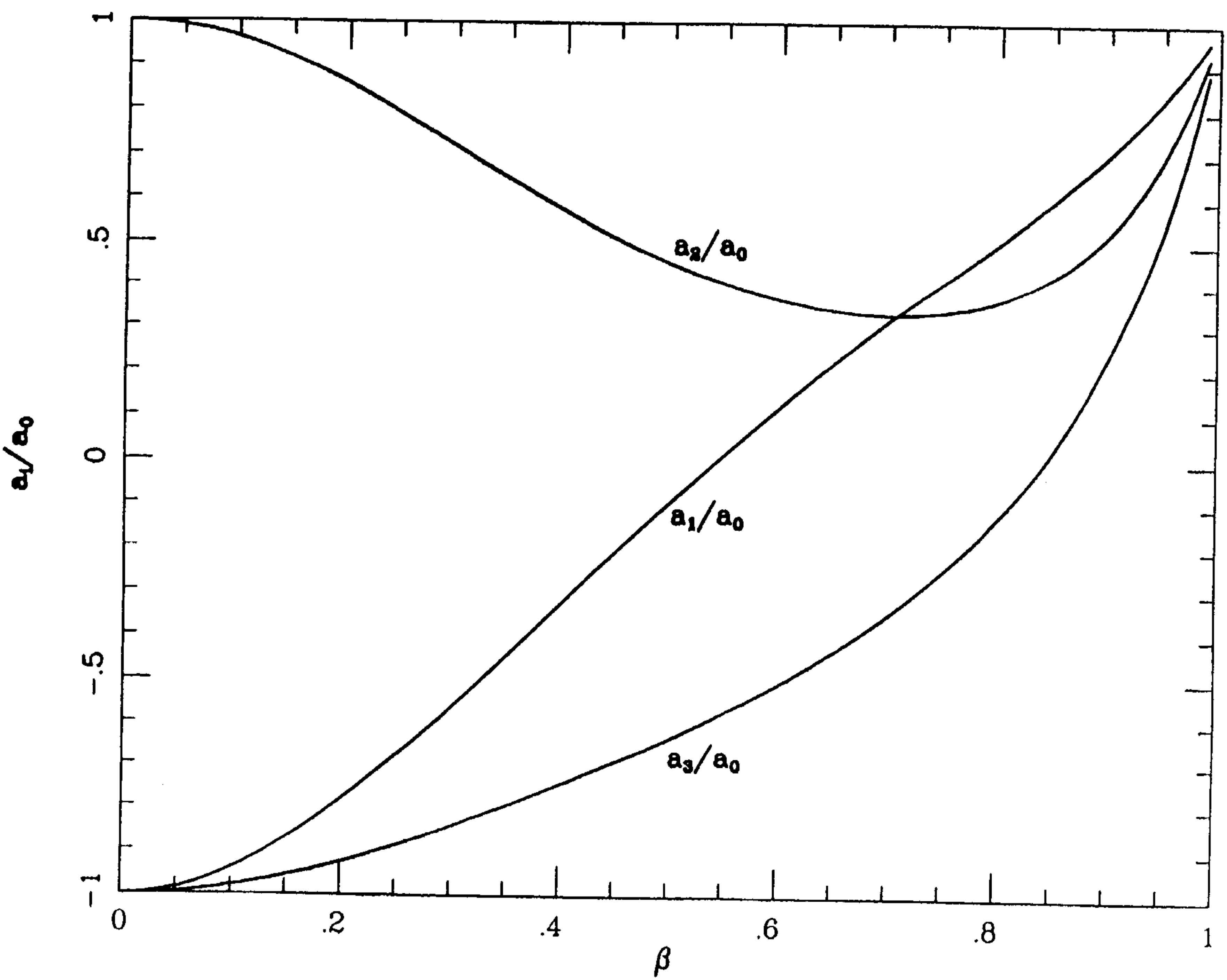

Fig.2